Computational Complexity of Air Travel Planning

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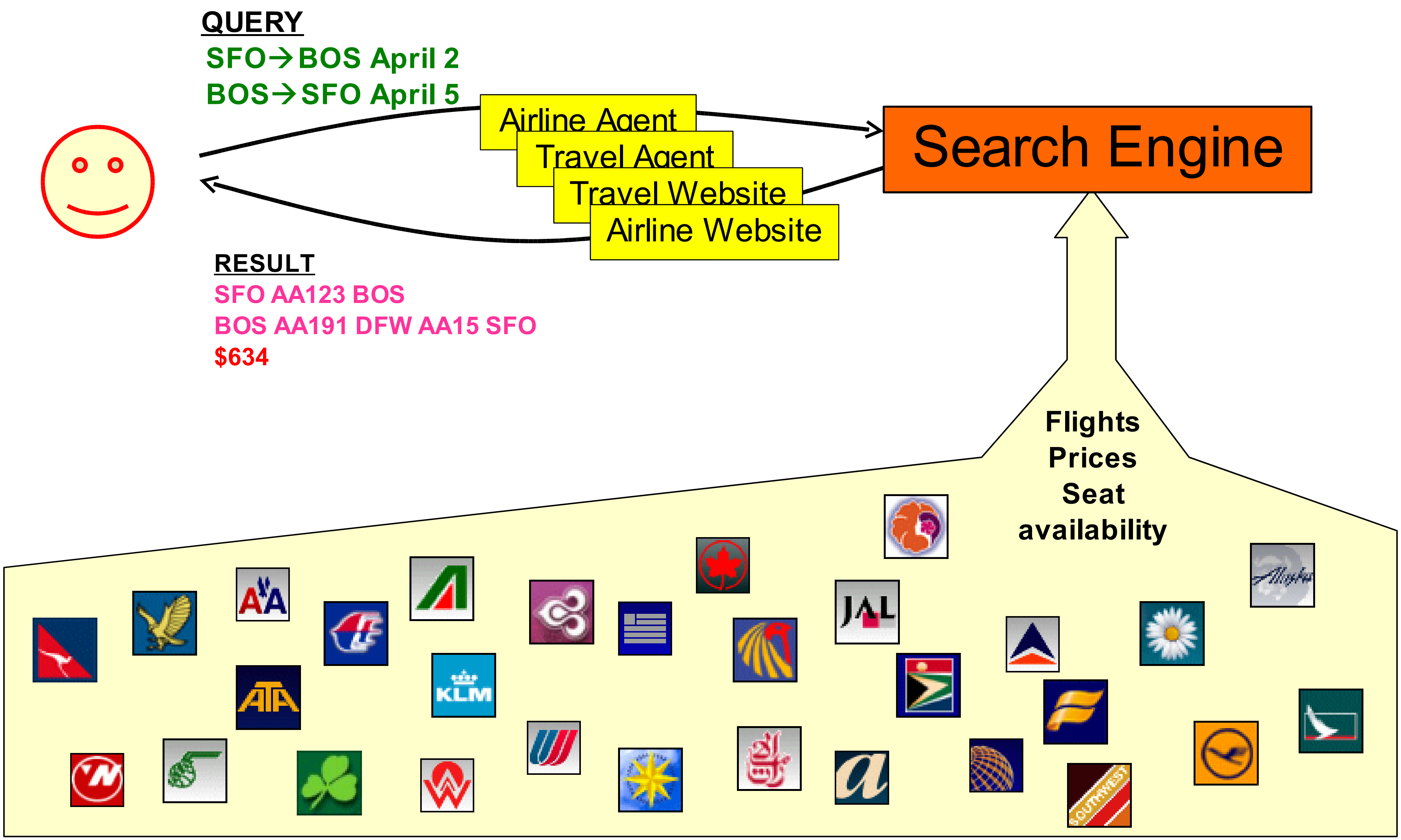
Air Travel Planning



Outline

- Introduction
- **Flights**
- How airline prices work
- Complexity of travel planning
- Demos
- Seat availability
- Further reading

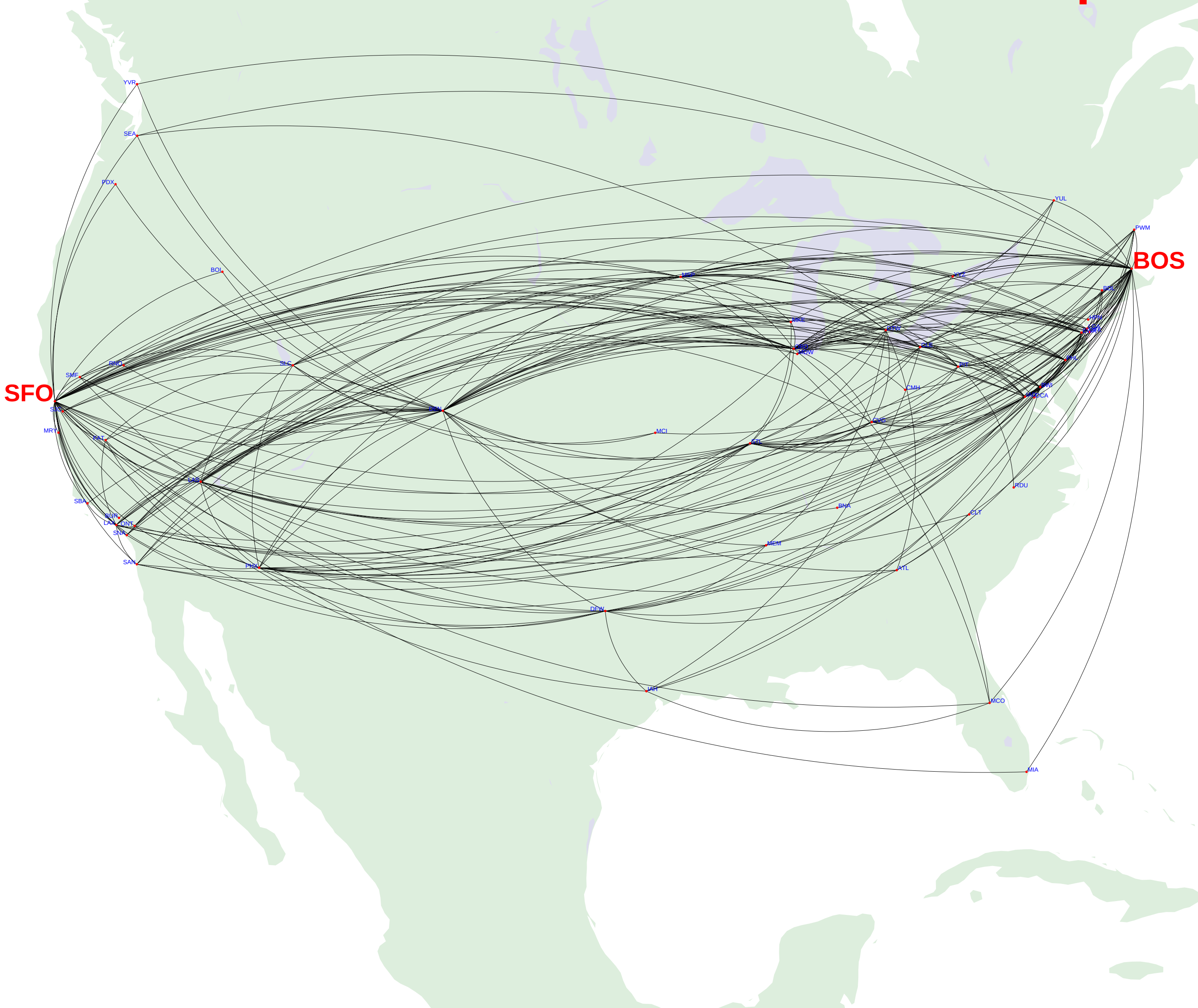
North American flights

The Flight Network

- 4000 airports served by commercial airlines
- Served by average of 4 airlines, connect to 8 others
- Weighted by # of departures, 22 airlines, 64 destinations
- Dominated by large airports
  - largest 1% (>4000 flights/day) have 40% of departures
  - largest 10% (>250 flights/day) have 85% of departures
  - reflects airlines' hub-and-spoke system
- Shortest path averages 3.5 in US, 5 worldwide (uniformly weighted)
- Diameter > 20

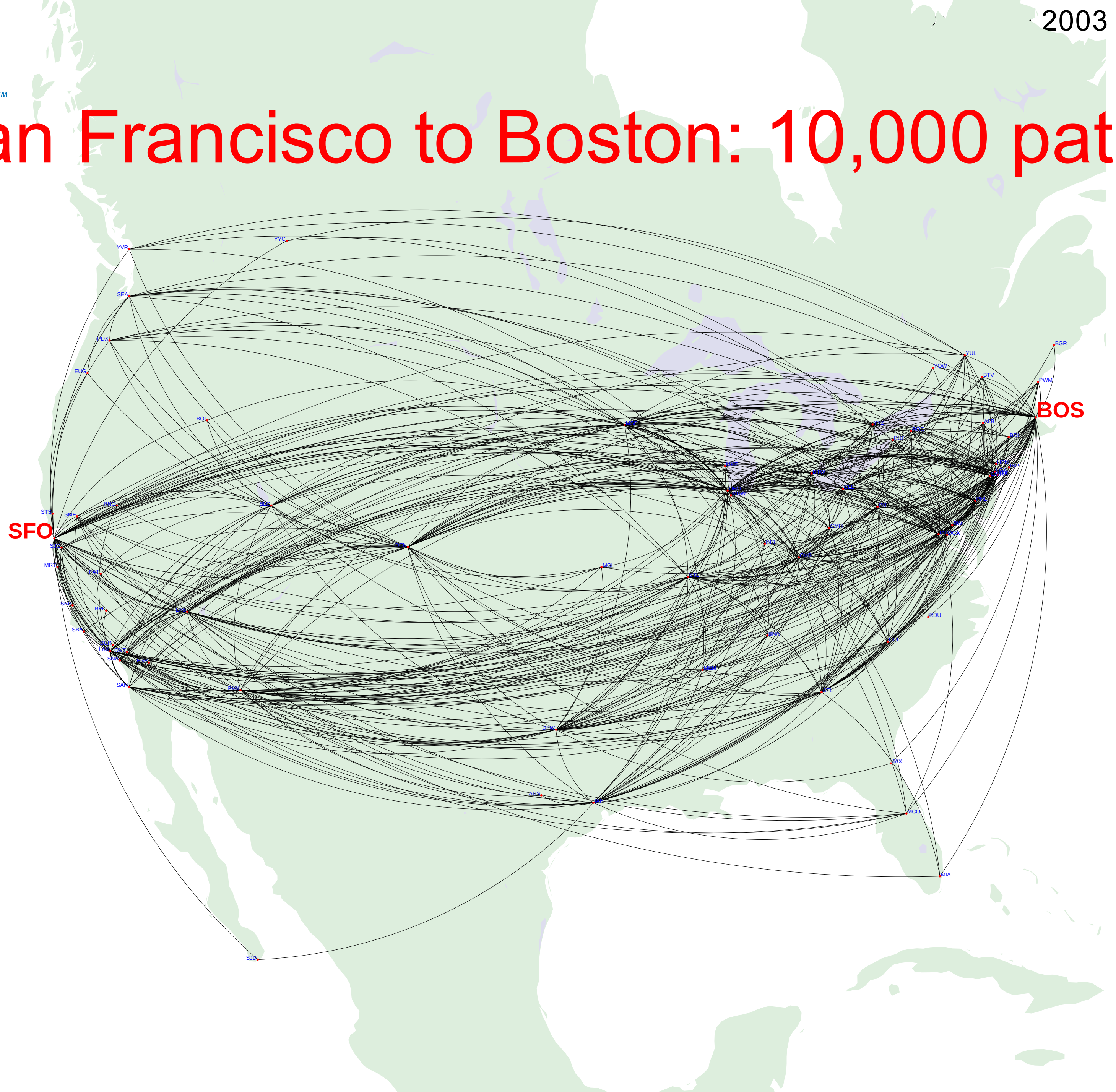
- 30,000,000 scheduled commercial flights per year – 1 per second
- 4000 – 10,000 planes in air, mostly large jets
- 700,000 passengers in the air
- 50% of flights within US and Canada

San Francisco to Boston: 2,000 paths



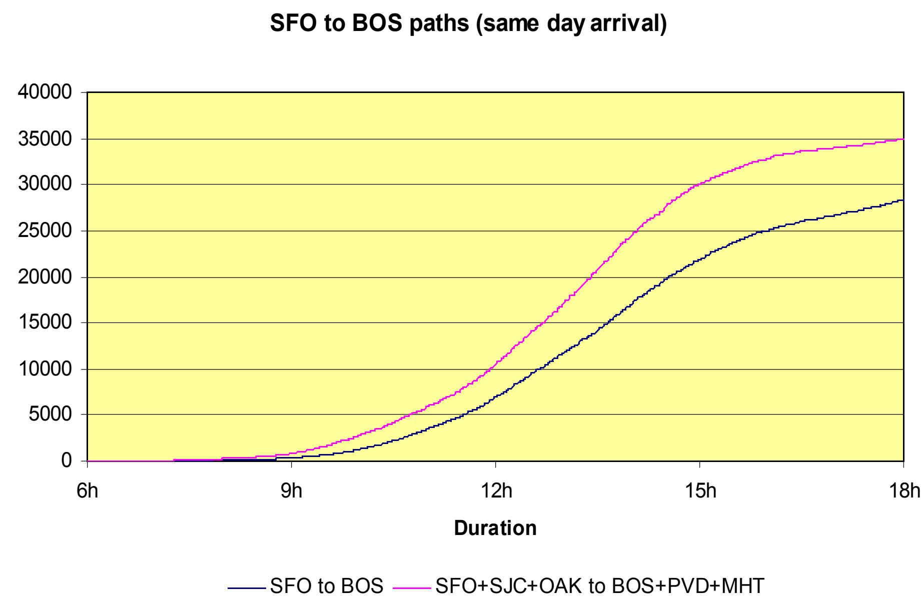
2003 ITA Software

San Francisco to Boston: 10,000 paths



Growth rate of # of paths

- Standard graph algorithms adequate to find one path
- Number of paths grows exponentially with duration or length



- Can't quickly enumerate all reasonable one-way itineraries; completely impractical to enumerate all round-trips
- Provably hard to use prices to inform selection

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Prices

- Almost all the difficulty in travel planning comes from prices
- ***Fare***: price for one-way travel between two cities (a *market*)

  AA BOS-SFO H14ESNR $436.28

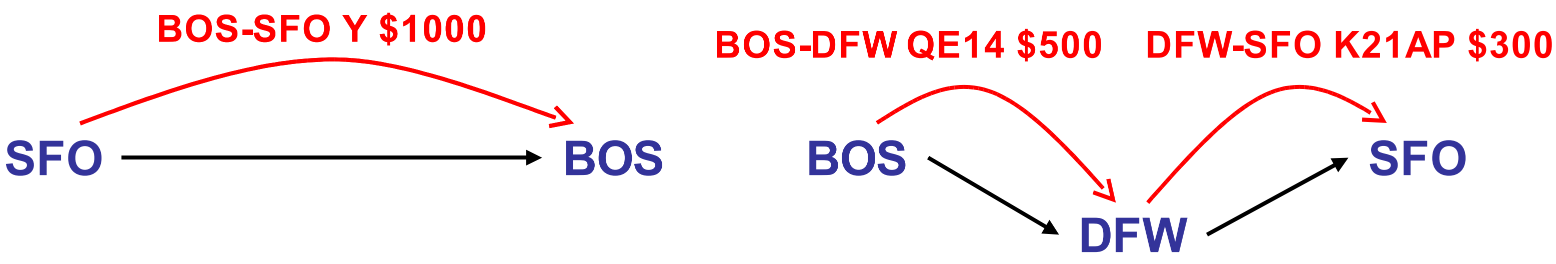
- Fare has ***rules*** restricting its use
- Axioms
  - Each flight must be covered (paid for) by exactly one fare
  - One fare may cover one or more (usually consecutive) flights
  - One or more fares are used to pay for a complete journey
- ***Fare component (FC)*** = fare + flights it covers

Fare components

$2000

Airline	City 1	City 2	Basis	Price
AA	BOS	SFO	Y	$1000
AA	BOS	DFW	QE14	$500
AA	DFW	SFO	K21AP	$300

Fare components

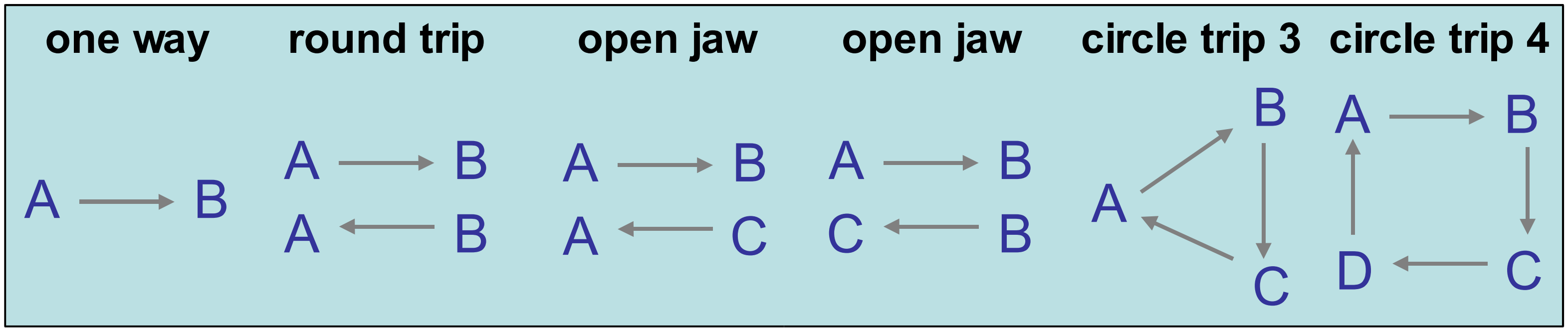


$1800

Airline	City 1	City 2	Basis	Price
AA	BOS	SFO	Y	$1000
AA	BOS	DFW	QE14	$500
AA	DFW	SFO	K21AP	$300

Priceable Units

- ***Priceable unit*** (PU) is a group of 1 to 4 fare components
  - restricted to one of several fixed geometries



- Ticket is built from one or more priceable units

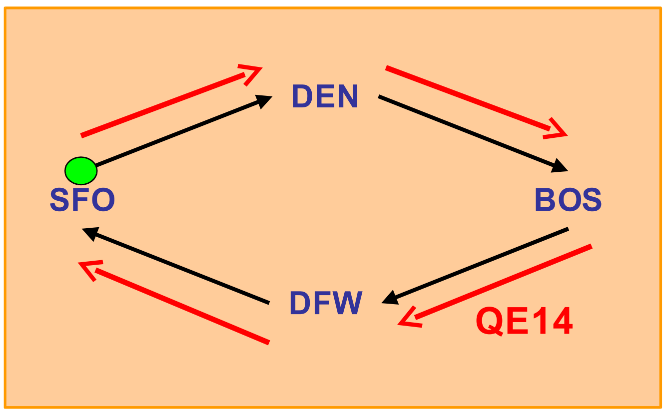
- PU is domain for fare rules such as minimum stay
  - *“Must be a Saturday night between departure of 1st flight in 1st fare component of PU and departure of 1st flight in last fare component”*

- Many cheap (“round trip”) fares do not participate in one-way PUs

Priceable Units

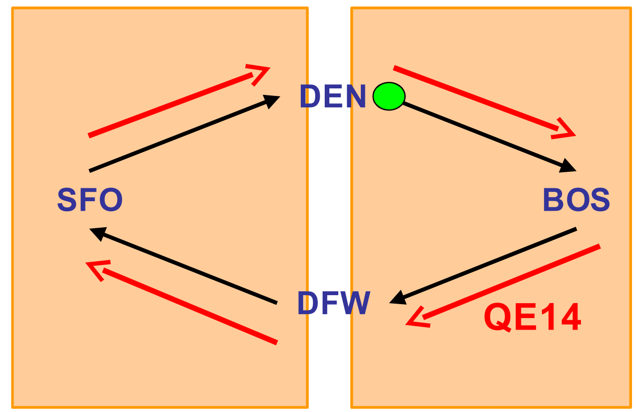
- Fare components may be grouped into priceable units in multiple ways
  - Affects the interpretation of fare rules

circle trip PU



14 AP: **purchase time to dep. SFO**

2 open jaw PUs



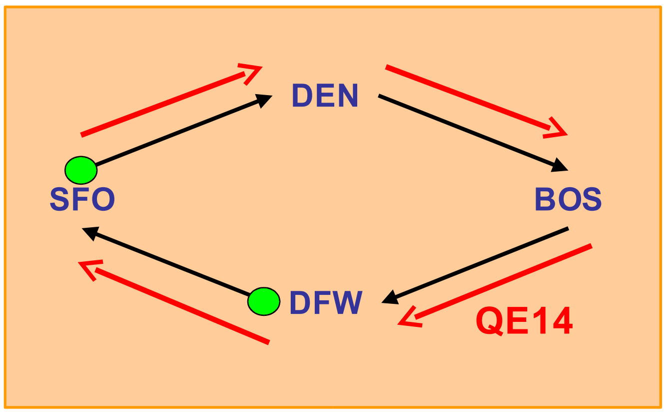
14 AP: **purchase time to dep. DEN**

QE14: **14 days advance purchase, Saturday-night stay**

Priceable Units

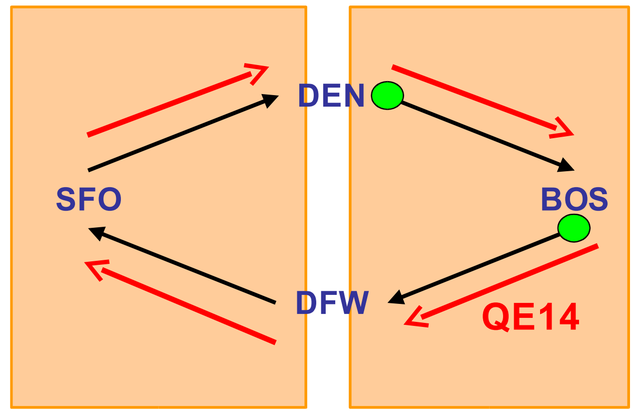
- Fare components may be grouped into priceable units in multiple ways
  - Affects the interpretation of fare rules

circle trip PU



14 AP: **purchase time to dep. SFO**
SAT: **dep. SFO to dep. DFW**

2 open jaw PUs



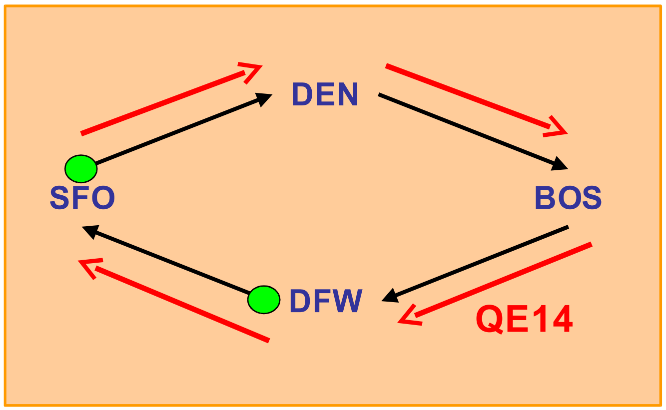
14 AP: **purchase time to dep. DEN**
SAT: **dep. DEN to dep. BOS**

QE14: **14 days advance purchase, Saturday-night stay**

Priceable Units

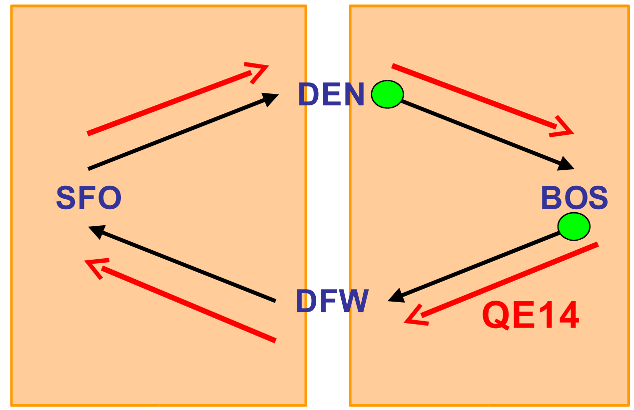
- Fare components may be grouped into priceable units in multiple ways
  - Affects the interpretation of fare rules

circle trip PU



14 AP: purchase time to dep. SFO
SAT: dep. SFO to dep. DFW

2 open jaw PUs



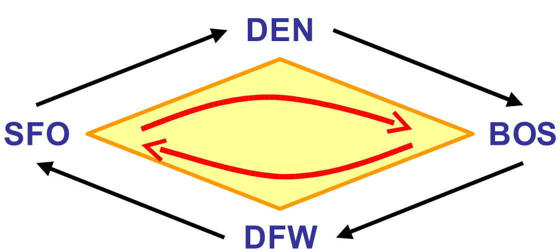
14 AP: purchase time to dep. DEN
SAT: dep. DEN to dep. BOS

Priceable units introduce long-distance flight & fare dependencies

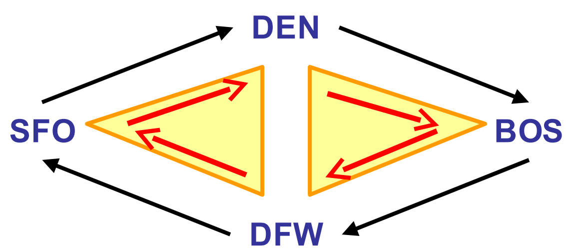
Priceable Units

- Flights can be broken into fare components and priceable units in many ways

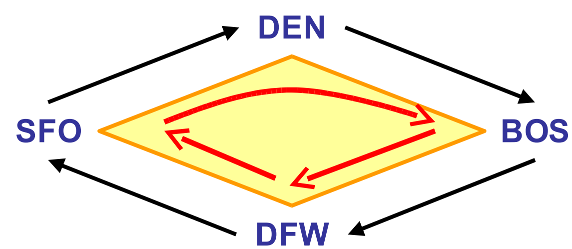
1 round trip



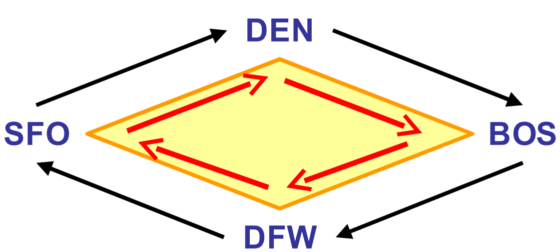
2 open jaws



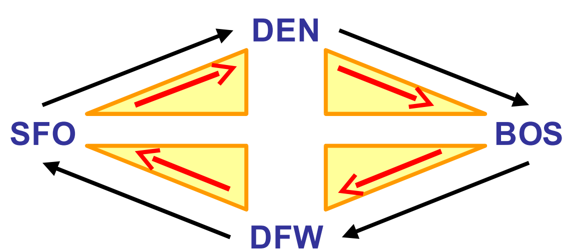
1 circle trip 3



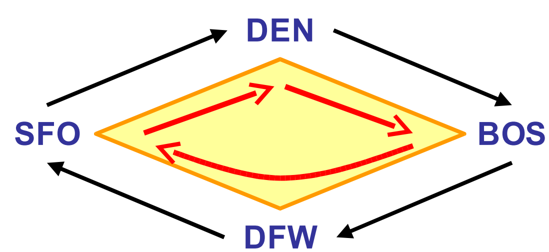
1 circle trip 4



4 one ways



1 circle trip 3



Fare Portfolio

- Airlines offer portfolio of fares at different prices in each market
  - From 5 to 500 fares (and more generated by macros)

BA BOS – LON							
AAP	£5663	HDWPXGB1	£578	MLF3CP	$377	R	£6142
B2	$653	HDXPXGB1	£558	MLF3IT	$377	VHF4CP	$502
DAP	£2951	HFWPX2	£435	MLFAM3FP	$378	VHF4IT	$502
DXRT	£3318	HFWPXGB1	£517	MLFAM3IT	$378	VYWAP2	£357
F1	£3469	HHWAPUS	$1063	MLWAPUS	$533	VYWAPGB1	£208
F1US	£543	HHWMTOW	$577	MLWSX7	£255	VYXAP2	£337
F2BA	£6608	HHWMTOW	$536	MLWSX8	£225	VYXAPGB1	£208
HAWPXGB1	£418	HHWPX2	£610	MLWSXGB1	£268	WUS	$1369
HAXPXGB1	£418	HHWPXGB1	£620	MLXAPUS	$473	Y	£837
HBWPXGB1	£516	HHXAPUS	$1003	MLXSX7	£235	Y2	£407
HBXPXGB1	£496	HHXMTOW	$515	MLXSX8	£225	YUS	$1369
HCWPXGB1	£437	HHXMTOW	$505	MLSXGB1	£268	**AND 239 MORE…**	
HCXPXGB1	£437	HHXPX2	£590	MQAPUS	$803		

Fare Rules

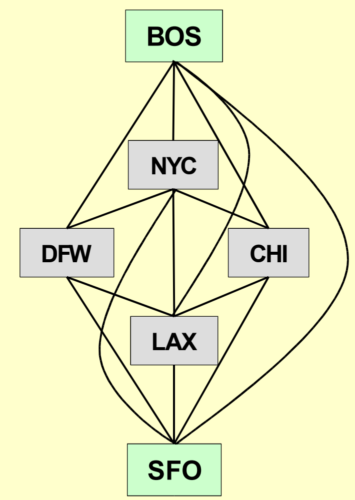
- Fare rules restrict use of each fare
  - Passengers
    - Age, nationality, occupation, employer, frequent flyer status
  - Fare component
    - Dates, times, locations, airlines, flights, duration of stops
  - Priceable unit
    - Types of priceable units (one way, round trip, open jaw, …)
    - Other fares in the priceable unit (airline and basis codes)
    - Dates, times, locations, airlines, flights, duration of stops
  - Journey
    - Fares and flights in other priceable units (airline and basis codes)
    - Other priceable unit geometries
  - Other
    - Purchase location and time

Sample Fare Rules

AA BOS-SFO H14ESNR $436.28

Rule	Details	Restricts
Tues or Weds	1st flight in FC must depart on Tues or Weds	FC flights
Surcharges	add $22.50 if BOS→SFO; add $20 if SFO→BOS	FC flights
14 days adv purchase	1st flight in PU must depart 14 days after reservations	PU flights
Saturday-night stay	complicated	PU flights
Combinability	all fares in PU must be on AA or TW; other restrictions; no OW PUs	PU fares PU geometry
Back-to-back	complicated	Other PU geometries
And much more	…	…

Routing



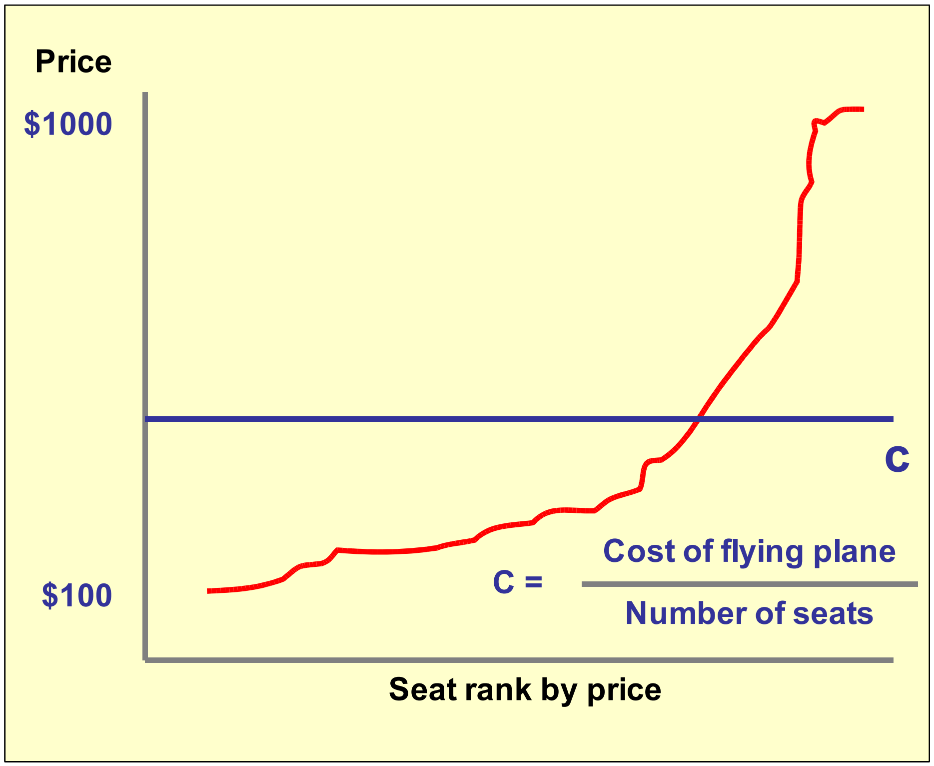
- Rules expressed in extremely complicated and baroque electronic language
  - ~1000 parameterized predicates
  - Very limited range of logical combinators
  - No quantifiers, variables, functions
  - Very limited expressive power

Summary: The Search Problem

- For a travel query, find the best solution
  - A set of flights that satisfies the travel query
  - A set of fares that covers all the flights exactly once
  - A partition of the fares into priceable units
  - For each fare, solution must satisfy fare's rules
    - Fare rules restrict
      - Flights in fare component
      - Flights and fares in other fare components of priceable unit
      - Priceable unit geometry
      - All flights and fares and priceable units in journey (less common)

Why this mess ? Variable pricing

$\forall p,\ p \cdot \text{demand}(p) < \text{Cost of flying plane}$



- Offer portfolio of fares at different prices
- Prevent the rich (business travelers) from using the cheap fares
  - Require advance purchase
  - Prohibit one-way priceable units
  - Require Saturday night stays
  - Prohibit nonstop routes
- Dynamically enable and disable fares according to demand

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- **Complexity of travel planning**
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Some Complexity Results

- Single fixed fare, fixed route, variable flights is NP-hard
- Fixed flights, fixed PUs, variable fares is NP-hard
- Fixed flights, fixed fares, variable PUs is NP-hard
- Full search is EXPSPACE-hard (simpler proof)
- Full search is undecidable (more difficult)

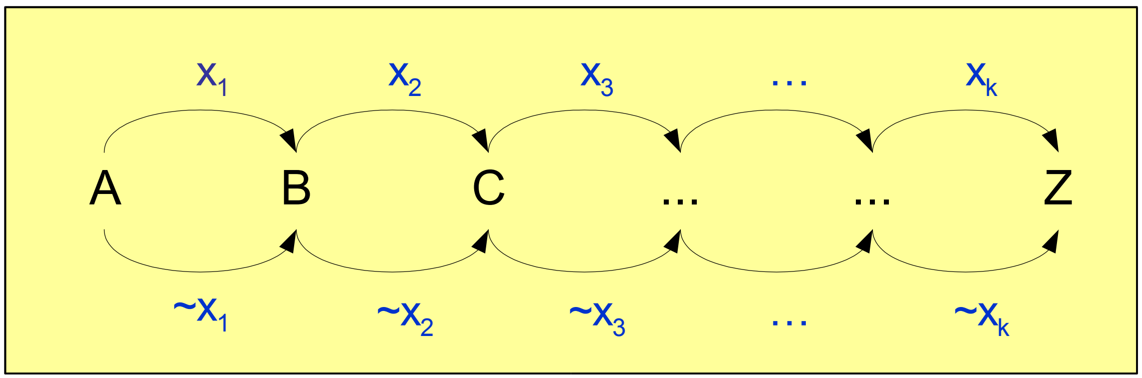
- Proofs rely only on fundamental parts of the pricing framework
- All proofs reduce standard problems to travel queries over specially constructed flight and fare databases

Single fare, fixed route is NP-hard

- One fixed fare, fixed route: only choice is flight number selection
- Reduce 3SAT (m clauses over k variables):

$$(x_1 \text{ or } x_2 \text{ or } \sim x_3) \wedge (x_2 \text{ or } \sim x_4 \text{ or } x_5) \wedge \ldots \wedge (\sim x_1 \text{ or } x_5 \text{ or } \sim x_k)$$

Flights:



Fare:

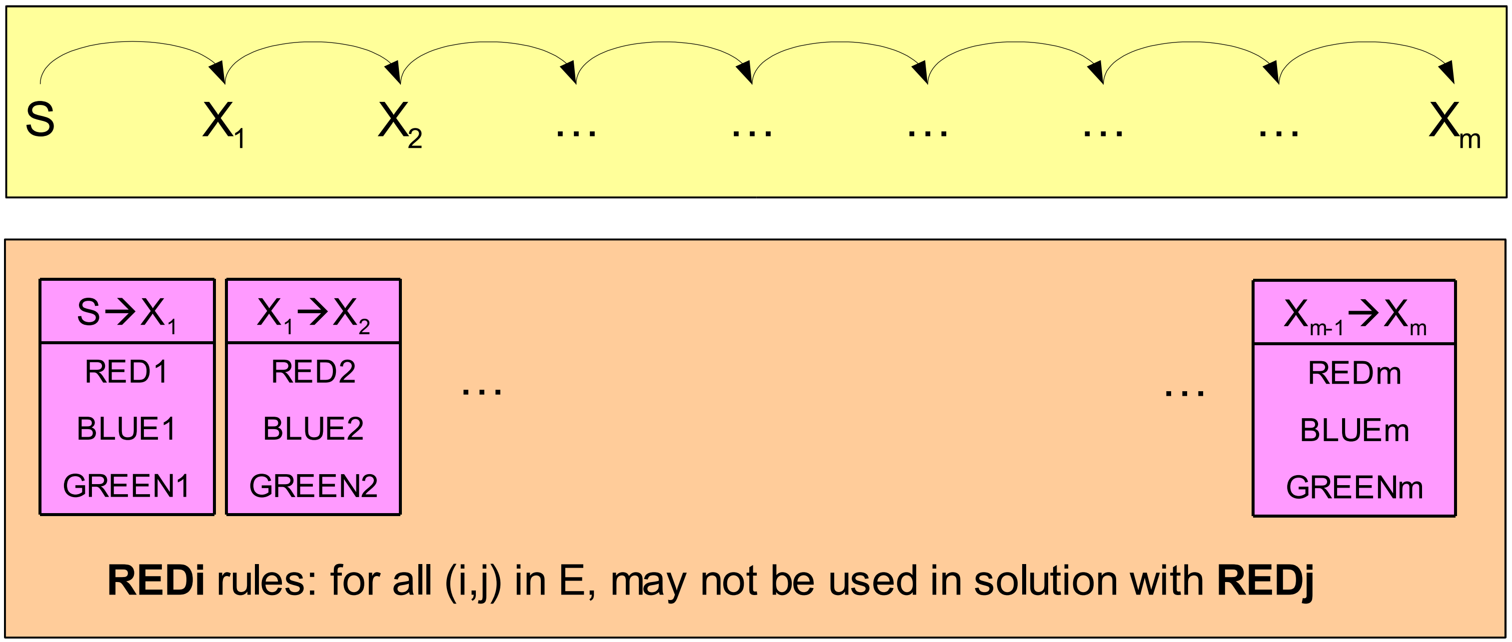
A→Z Rules = If $\sim x_1$ and $\sim x_2$ and x_3 then FAIL
If $\sim x_2$ and x_4 and $\sim x_5$ then FAIL
…
If x_1 and $\sim x_5$ and x_k then FAIL
Else PASS

Fixed flights, variable fares is NP-hard

- Flights are fixed: choice is over fares for each flight
- Reduce k-Color (m vertices)
- Fares can restrict fare basis codes of all other fares in solution

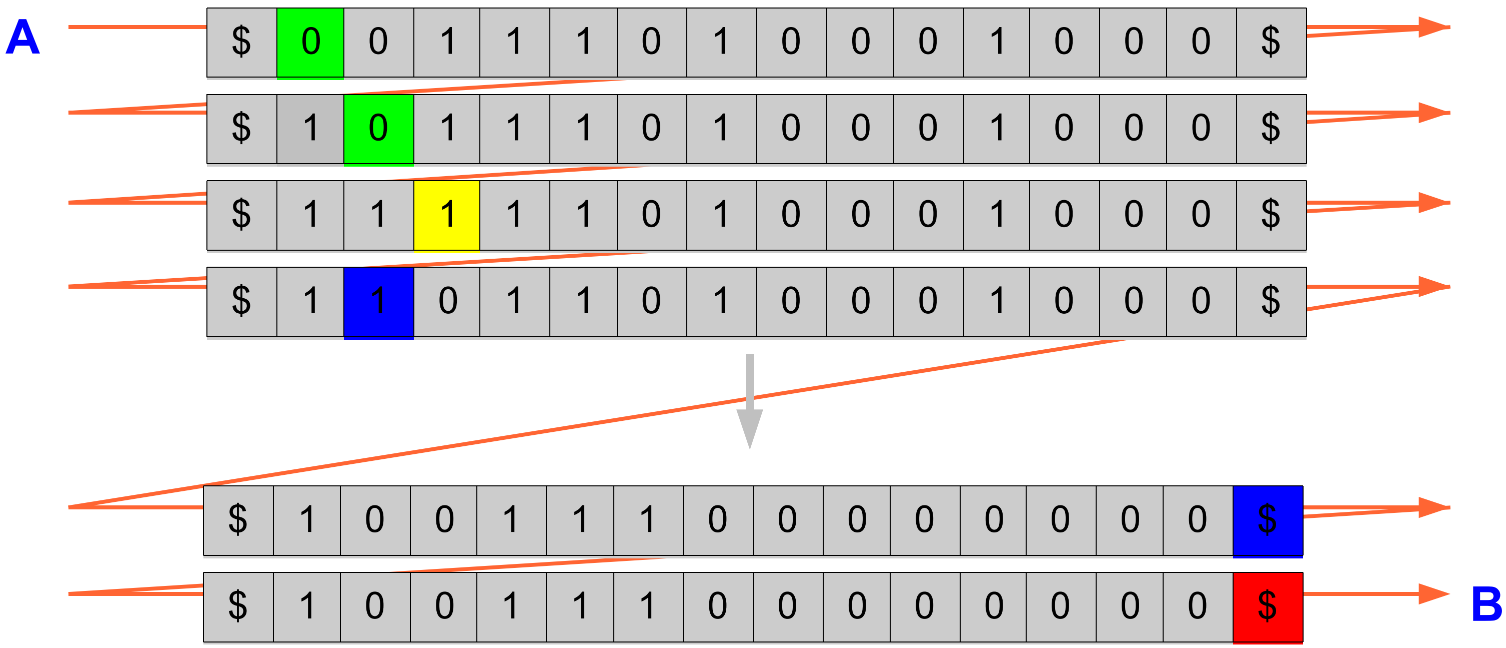
Flights:

Fares:



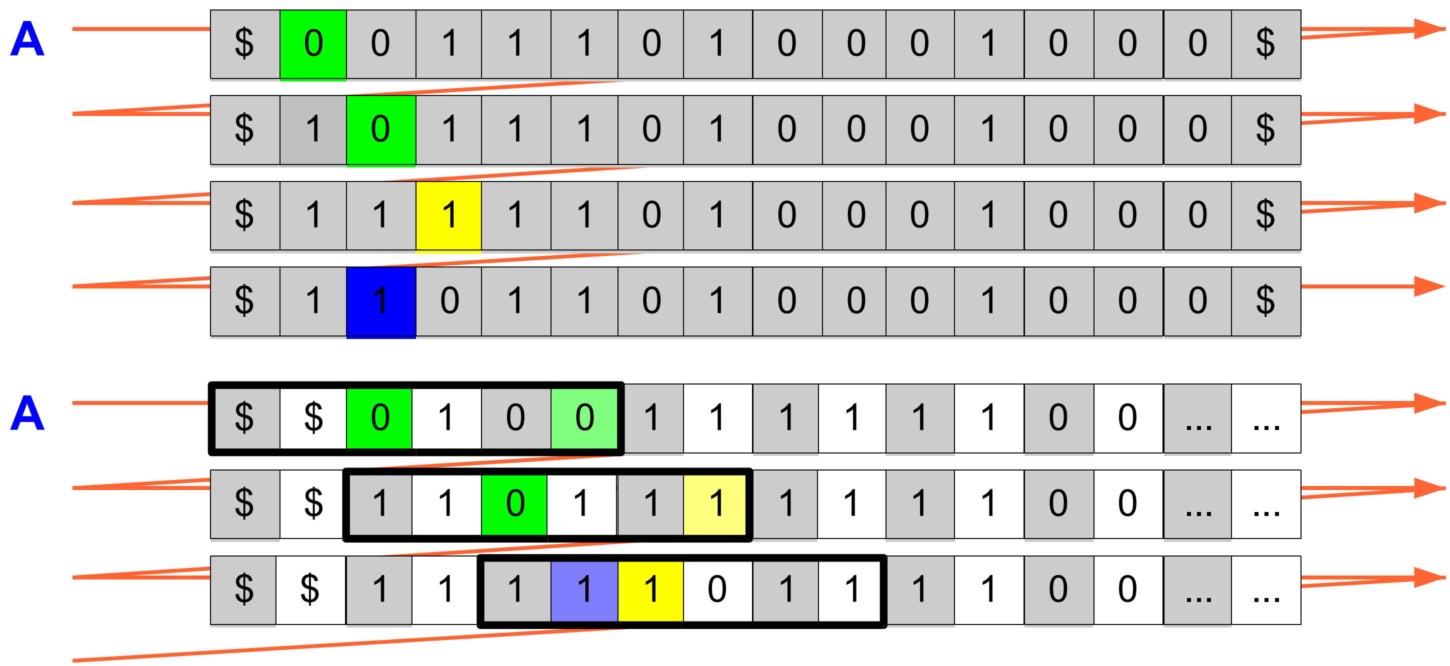
Full search is EXPSPACE-hard

- Simulate Turing Machine with exponential size tape
  – Flight represents a tape cell's contents at a particular time, including head position and state (all encoded into flight number)
- Trip flights from A to B encode entire history of Turing Machine's execution
- Final flight to destination B can only be taken from accept state



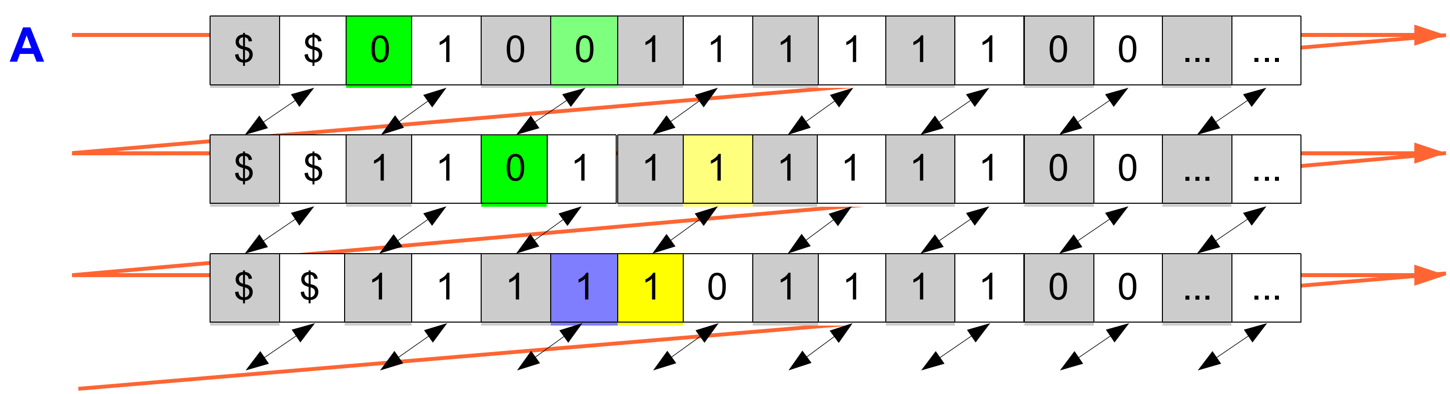
One-step consistency

- Key is that one time step and the next are related by a “regular relation”: can be expressed by a finite-state transducer
  - $\$:\$\ (0{:}0|1{:}1)^*\ (L_1{:}A_1\ Q_1{:}B_1\ R_1{:}C_1|L_2{:}A_2\ Q_2{:}B_2\ R_2{:}C_2|\ldots)\ (0{:}0|1{:}1)^*\ \$:\$$
  - Writing, moving and state transitions expressed by small table of triples
- If we collapse into one sequence of alternating symbols, can be expressed using FSM
  - $\$\$\ (00|11)^*\ (L_1A_1\ Q_1B_1\ R_1C_1|L_2A_2\ Q_2B_2\ R_2C_2|\ldots)\ (00|11)^*\ \$\$$



Multi-step consistency

- Now one-step transitions are encoded within a time step by FSM flight graph
- To ensure multi-step consistency, need to enforce equality between cells on a diagonal
  - Implemented using round-trip priceable-units that enforce same-flight-number restrictions on outbound flight and return flight
  - Key issue is ensuring that right cells are paired; implemented using minimum and maximum stay restrictions: min stay = max stay = TAPE-LENGTH * 2 - 1
  - EXP-SPACE limit comes from encoding of min/max stay: n bits encodes 2^n length



Some Details

- Finite-state machine encoding must be of size polynomial in the input, but allow for exponentially many flights
  - Electronic flight formats permit one to say “Flight X leaves *every* day at 5pm”
  - Encoding size is thus governed only by representation of input and number of transition triples
    - Polynomial in input (TM specification and input tape)
    - TM specification bounded at a small number if one encodes a Universal TM and writes the program on the tape
  - Minimum connection time (MCTs) tables make it easy to encode FSM
    - MCTs are per-airport specifications of whether one can connect between two flights with specified flight numbers, and if one can, minimum time that must be allowed
- Can simulate non-deterministic TMs because their permitted transitions are just as easily encoded using an FSM as deterministic TMs
- Solutions are big
  - For EXPSPACE, no limit on size of solution because no limit on # of steps
  - If polynomial limit is placed on solution size, then can simulate polynomial-sized tape for polynomial number of steps: NP-hard
  - ITA Software's engine can run a TM over a tape of size 10 to 20 for 10 to 20 steps
- No need to specify input tape: can let system search over all possible input tapes

Full search is unsolvable

- **Air travel planning is unsolvable for certain inputs**
- Reduce the Diophantine decision problem

$$\{ x_1 \ldots x_n \in Z^+ \mid P(x_1 \ldots x_n) = 0 \} = \varnothing \ ?$$

- Value x represented by |X|, the number of X fares in solution
- Example:

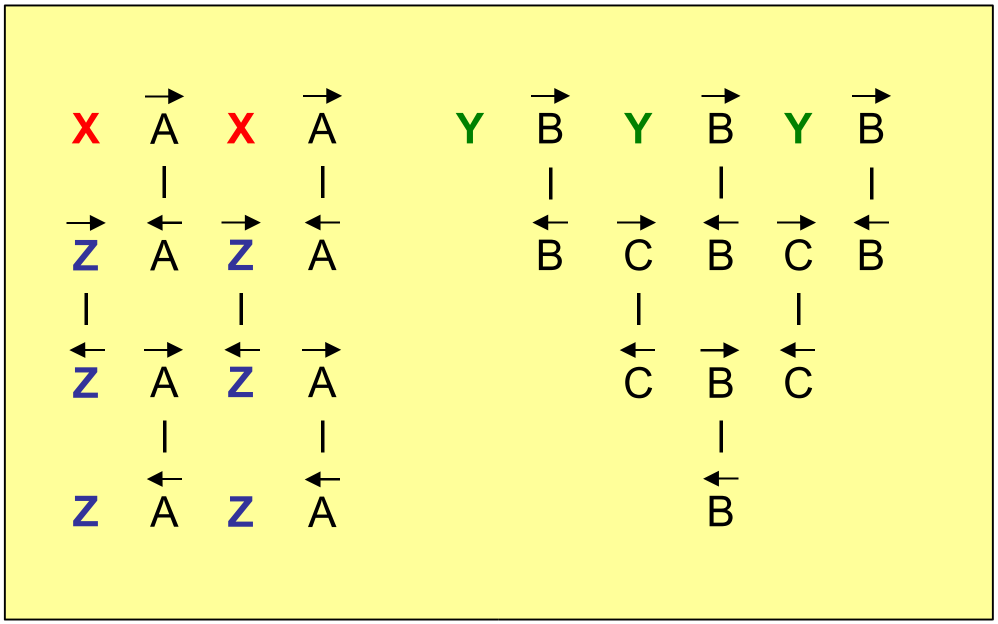
$$ab^2 - 3b = 0$$

Constrain solution to form $A^+B^+C^+P^+N^+$, where $|C|=|B||B|$, $|P|=|A||C|$, $|N|=3|B|$
- $|P|$ is sum of positive terms, $|N|$ is sum of negative terms
- Enforce $|P| = |N|$ using round trip priceable units

- Key challenge is enforcing multiplication: $|Z|=|X||Y|$

Unary multiplication with fares

- Example: *2·3 = 6* (|X|=2, |Y|=3, |Z|=6)



Trip = S(OE)*(O’ | OE’)

$S = (X\vec{A})^+(Y\vec{B})^+$

$O = (\vec{Z}\overleftarrow{A})^+(\overleftrightarrow{BC})^+\overleftarrow{B}$

$E = (\overleftrightarrow{ZA})^+(\overleftrightarrow{CB})^*\overleftarrow{C}$

$O' = (Z\overleftarrow{A})^+\overleftarrow{B}$ $E' = \overleftarrow{Z}^+\overleftarrow{C}$

- Structure lets “back to back” restriction work around time limits in EXPSPACE-hard proof; details are complicated

Complexity Review

- Even the most basic subproblems are provably hard

- Proofs reflect the real algorithmic challenges we have experienced
- Complexity proofs are harder than they look
  - electronic format for fare rules is complicated but very limited
- Heuristics risky: airlines can change their fare and rule structures instantaneously; sometimes deliberately complicate space
- Order-of-growth is a serious issue:
  - 30,000,000 flights in database
  - 150,000,000 fares in database
  - 10,000 to100,000,000 flight combinations for a round-trip
  - 10,000 to100,000,000 fare combinations for each flight combo
  - much worse for multiple passengers

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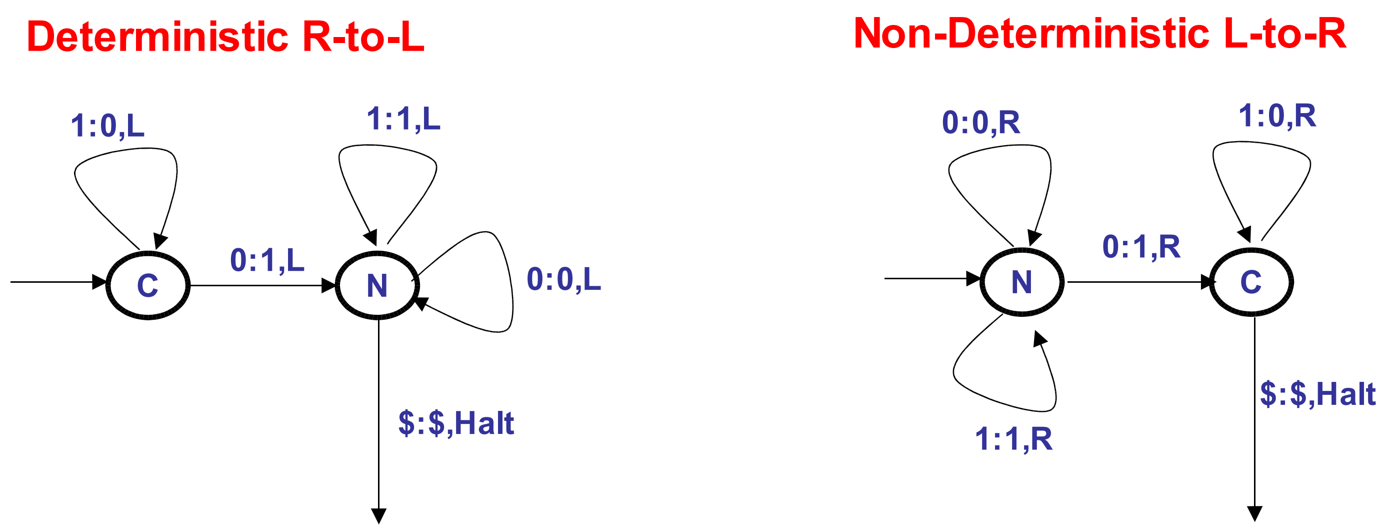
Turing Machine Simulations

- Actually write code to translate programs into industry-standard fares and rules
- Run on ITA Software's production servers with unmodified code

- What can we handle in practice?
  - Non-deterministic Turing Machines
    - Search all inputs at once
  - With production code settings
    - Max tape length with 0/1 alphabet ~20
    - Max execution steps ~20
    - Max ~10 states
    - Takes about 1 second to run
  - Thus, e.g, small problems in NP
  - Standard Shannon/Minsky alphabet/state tradeoff theorems apply

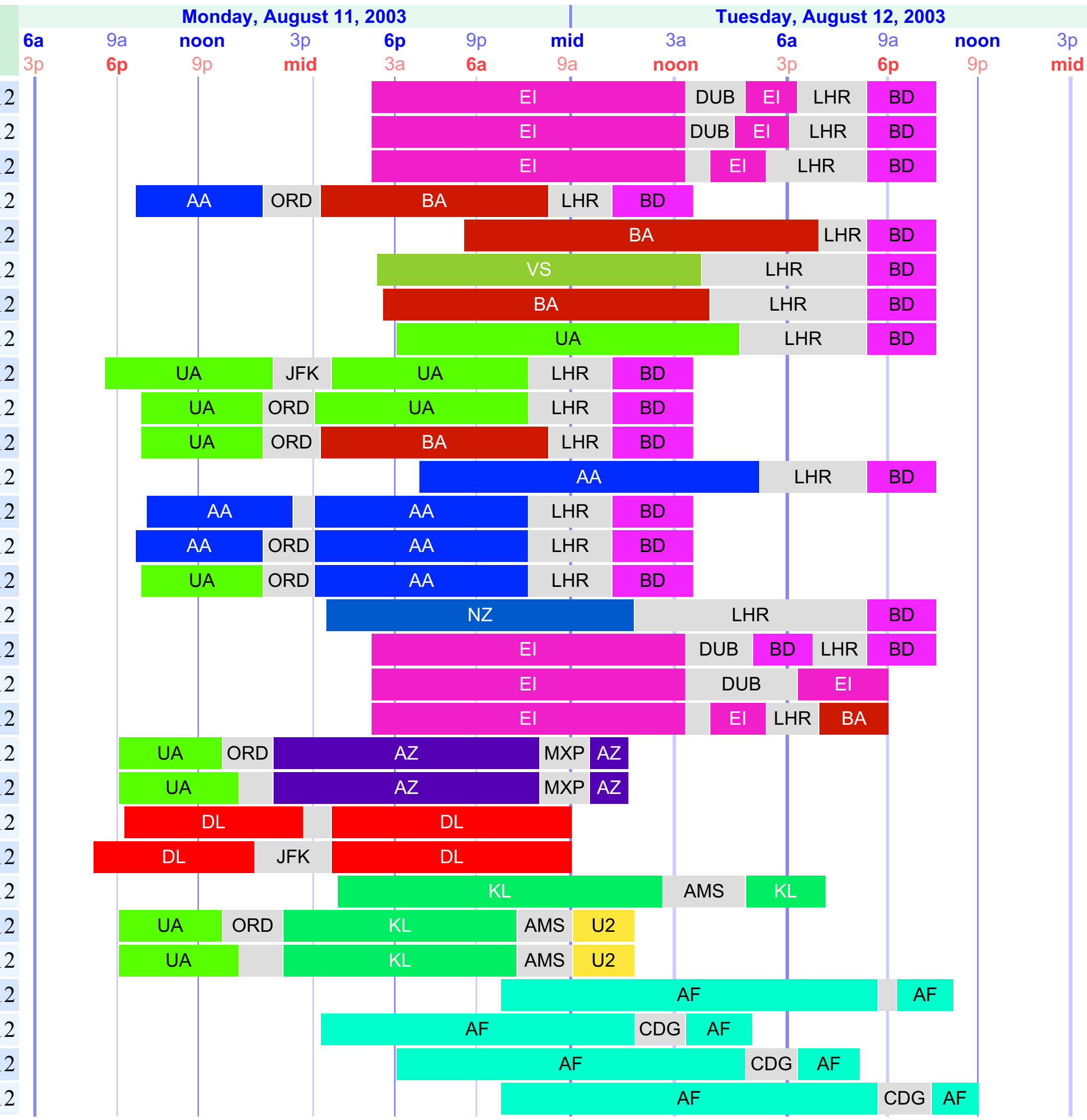
Right to Left Increment

- Right-to-left boolean increment-by-1 machine is 2-state DTM (DFST)
- Left-to-right boolean increment-by-1 machine is 2-state NTM (NFST)
- Set prices of “1” fares to reflect bit position
  - 1.00$*$2^i$ input tape, 0.01$*$2^i$ output tape

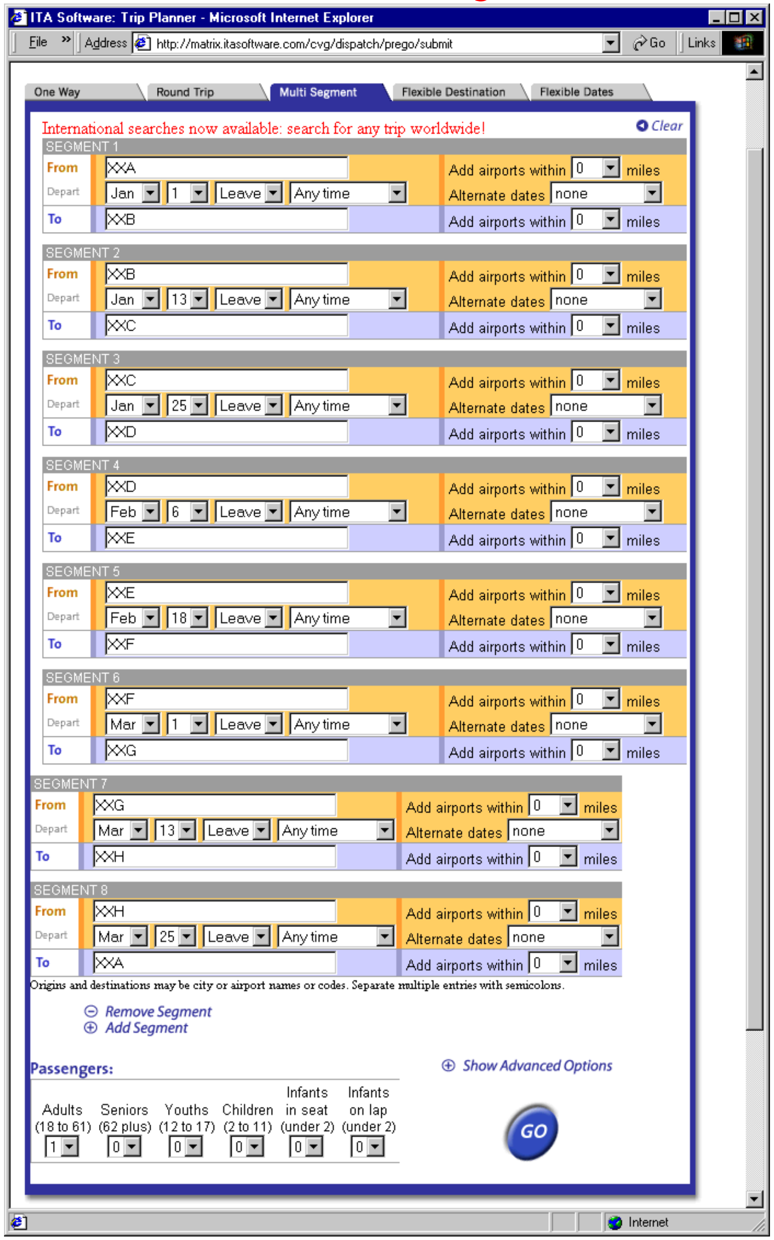


Graphical Presentation

Cost	From/To	Date
$907.21	LAX to NCE	Mon-Tue, Aug 11-12
$907.21	LAX to NCE	Mon-Tue, Aug 11-12
$907.21	LAX to NCE	Mon-Tue, Aug 11-12
$1045.71	LAX to NCE	Mon-Tue, Aug 11-12
$1049.21	LAX to NCE	Mon-Tue, Aug 11-12
$1049.21	LAX to NCE	Mon-Tue, Aug 11-12
$1049.21	LAX to NCE	Mon-Tue, Aug 11-12
$1053.21	LAX to NCE	Mon-Tue, Aug 11-12
$1053.21	LAX to NCE	Mon-Tue, Aug 11-12
$1057.71	LAX to NCE	Mon-Tue, Aug 11-12
$1069.71	LAX to NCE	Mon-Tue, Aug 11-12
$1074.21	LAX to NCE	Mon-Tue, Aug 11-12
$1078.71	LAX to NCE	Mon-Tue, Aug 11-12
$1078.71	LAX to NCE	Mon-Tue, Aug 11-12
$1098.71	LAX to NCE	Mon-Tue, Aug 11-12
$1155.36	LAX to NCE	Mon-Tue, Aug 11-12
$1196.21	LAX to NCE	Mon-Tue, Aug 11-12
$1300.90	LAX to NCE	Mon-Tue, Aug 11-12
$1409.21	LAX to NCE	Mon-Tue, Aug 11-12
$1556.56	LAX to NCE	Mon-Tue, Aug 11-12
$1556.56	LAX to NCE	Mon-Tue, Aug 11-12
$1621.90	LAX to NCE	Mon-Tue, Aug 11-12
$1621.90	LAX to NCE	Mon-Tue, Aug 11-12
$1624.83	LAX to NCE	Mon-Tue, Aug 11-12
$1629.33	LAX to NCE	Mon-Tue, Aug 11-12
$1629.33	LAX to NCE	Mon-Tue, Aug 11-12
$1629.52	LAX to NCE	Mon-Tue, Aug 11-12
$1629.52	LAX to NCE	Mon-Tue, Aug 11-12
$1629.52	LAX to NCE	Mon-Tue, Aug 11-12
$1629.52	LAX to NCE	Mon-Tue, Aug 11-12



Query


ITA Software: Trip Planner - Microsoft Internet Explorer
http://matrix.itasoftware.com/cvg/dispatch/prego/submit
One Way
Round Trip
Multi Segment
Flexible Destination
Flexible Dates
International searches now available: search for any trip worldwide!
Clear
SEGMENT 1
From XXA
Depart Jan 1 Leave Any time
To XXB
SEGMENT 2
From XXB
Depart Jan 13 Leave Any time
To XXC
SEGMENT 3
From XXC
Depart Jan 25 Leave Any time
To XXD
SEGMENT 4
From XXD
Depart Feb 6 Leave Any time
To XXE
SEGMENT 5
From XXE
Depart Feb 18 Leave Any time
To XXF
SEGMENT 6
From XXF
Depart Mar 1 Leave Any time
To XXG
SEGMENT 7
From XXG
Depart Mar 13 Leave Any time
To XXH
SEGMENT 8
From XXH
Depart Mar 25 Leave Any time
To XXA
Add airports within 0 miles
Alternate dates none
Origins and destinations may be city or airport names or codes. Separate multiple entries with semicolons.
Remove Segment
Add Segment
Show Advanced Options
Passengers:
Adults (18 to 61) 1
Seniors (62 plus) 0
Youths (12 to 17) 0
Children (2 to 11) 0
Infants in seat (under 2) 0
Infants on lap (under 2) 0
GO
Internet

Incrementer Results

Cost	Slice	From/To	Date	day of dep.		next day		+ 2 days		+ 3 days		+ 4 days		+ 5 days	
			mid	noon	mid	noon	mid	noon	mid	noon	mid	noon	mid	noon	mid
$55.56	0	XXA to XXB	Sat-Thur, Nov 1-6	N1	T1	T1	N1	T0	T0	T1	T1	T1	T1	T1	T1
	1	XXB to XXC	Fri-Wed, Nov 7-12	T1	T1	N1	T1	T0	N0	T1	T1	T1	T1	T1	T1
	2	XXC to XXD	Thur-Tue, Nov 13-18	T1	T1	T1	T1	N0	T1	T1	C1	T1	T1	T1	T1
	3	XXD to XXE	Wed-Mon, Nov 19-24	T1	T1	T1	T1	T1	T1	C1	T0	T1	C1	T1	T1
	4	XXE to XXF	Tue-Sun, Nov 25-30	T1	T1	T1	T1	T1	T1	T0	T0	C1	T0	T1	C1
	5	XXF to XXG	Mon-Sat, Dec 1-6	T1	T1	T1	T1	T1	T1	T0	T0	T0	T0	C1	T0
	6	XXG to XXA	Sun-Fri, Dec 7-12	T1	T1	T1	T1	T1	T1	T0	T0	T0	T0	T0	T0
$56.57	0	XXA to XXB	Sat-Thur, Nov 1-6	N1	T1	T1	N1	T1	T1	T0	T0	T0	T0	T0	T0
	1	XXB to XXC	Fri-Wed, Nov 7-12	T1	T1	N1	T1	T1	N1	T0	T0	T0	T0	T0	T0
	2	XXC to XXD	Thur-Tue, Nov 13-18	T1	T1	T1	T1	N1	T1	T0	N0	T0	T0	T0	T0
	3	XXD to XXE	Wed-Mon, Nov 19-24	T1	T1	T1	T1	T1	T1	N0	T0	T0	N0	T0	T0
	4	XXE to XXF	Tue-Sun, Nov 25-30	T1	T1	T1	T1	T1	T1	T0	T0	N0	T0	T0	N0
	5	XXF to XXG	Mon-Sat, Dec 1-6	T1	T1	T1	T1	T1	T1	T0	T0	T0	T0	N0	T1
	6	XXG to XXA	Sun-Fri, Dec 7-12	T1	T1	T1	T1	T1	T1	T0	T0	T0	T0	T1	T1
$57.58	0	XXA to XXB	Sat-Thur, Nov 1-6	N1	T1	T1	N1	T1	T1	T0	T0	T0	T0	T1	T1
	1	XXB to XXC	Fri-Wed, Nov 7-12	T1	T1	N1	T1	T1	N1	T0	T0	T0	T0	T1	T1
	2	XXC to XXD	Thur-Tue, Nov 13-18	T1	T1	T1	T1	N1	T1	T0	N0	T0	T0	T1	T1
	3	XXD to XXE	Wed-Mon, Nov 19-24	T1	T1	T1	T1	T1	T1	N0	T0	T0	N0	T1	T1
	4	XXE to XXF	Tue-Sun, Nov 25-30	T1	T1	T1	T1	T1	T1	T0	T0	N0	T1	T1	C1
	5	XXF to XXG	Mon-Sat, Dec 1-6	T1	T1	T1	T1	T1	T1	T0	T0	T1	T1	C1	T0
	6	XXG to XXA	Sun-Fri, Dec 7-12	T1	T1	T1	T1	T1	T1	T0	T0	T1	T1	T0	T0

Bit rotatation

Cost	Slice	From/To	Date	day of departure		next day		2 days after		3 days after		4 days after	
				mid noon	mid	noon	mid	noon	mid	noon	mid	noon	mid
$10.05	0	XXA to XXB	Sat-Wed, Nov 1-5	W0	T0	T1	W1	T0	T0	T1	T1	T0	T0
	1	XXB to XXC	Thur-Mon, Nov 6-10	T0	T0	W1	T0	T0	X0	T1	T1	T0	T0
	2	XXC to XXD	Tue-Sat, Nov 11-15	T0	T0	T0	T0	X0	T1	T1	W1	T0	T0
	3	XXD to XXE	Sun-Thur, Nov 16-20	T0	T0	T0	T0	T1	T1	W1	T0	T0	X0
	4	XXE to XXF	Fri-Tue, Nov 21-25	T0	T0	T0	T0	T1	T1	T0	Y0	X0	T1
	5	XXF to XXG	Wed-Sun, Nov 26-30	T0	T0	T0	T0	T1	Y1	Y0	T0	T1	T1
	6	XXG to XXH	Mon-Fri, Dec 1-5	T0	T0	T0	Y0	Y1	T1	T0	T0	T1	T1
	7	XXH to XXI	Sat-Wed, Dec 6-10	T0	Y0	Y0	T0	T1	T1	T0	T0	T1	T1
	8	XXI to XXJ	Thur-Mon, Dec 11-15	Y0	T0	T0	T0	T1	T1	T0	T0	T1	T1
	9	XXJ to XXA	Tue-Sat, Dec 16-20	T0	T0	T0	T0	T1	T1	T0	T0	T1	T1
$11.21	0	XXA to XXB	Sat-Wed, Nov 1-5	W0	T0	T1	W1	T0	T0	T1	T1	T1	T1
	1	XXB to XXC	Thur-Mon, Nov 6-10	T0	T0	W1	T0	T0	X0	T1	T1	T1	T1
	2	XXC to XXD	Tue-Sat, Nov 11-15	T0	T0	T0	T0	X0	T1	T1	W1	T1	T1
	3	XXD to XXE	Sun-Thur, Nov 16-20	T0	T0	T0	T0	T1	T1	W1	T0	T1	X1
	4	XXE to XXF	Fri-Tue, Nov 21-25	T0	T0	T0	T0	T1	T1	T0	Z0	X1	T1
	5	XXF to XXG	Wed-Sun, Nov 26-30	T0	T0	T0	T0	T1	Z1	Z0	T0	T1	T1
	6	XXG to XXH	Mon-Fri, Dec 1-5	T0	T0	T0	Z0	Z1	T1	T0	T0	T1	T1
	7	XXH to XXI	Sat-Wed, Dec 6-10	T0	Z0	Z0	T0	T1	T1	T0	T0	T1	T1
	8	XXI to XXJ	Thur-Mon, Dec 11-15	Z0	T1	T0	T0	T1	T1	T0	T0	T1	T1
	9	XXJ to XXA	Tue-Sat, Dec 16-20	T1	T1	T0	T0	T1	T1	T0	T0	T1	T1

Multiplication

- Implement multiplication circuits
  - Both unary and binary multiplication
  - Unary is core of undecidability proof
  - Not based on TMs, but just as with TM simulation, round-trip PUs used to encode finite-state transducers

- Multiply: solutions that start with flight sequences “17” and “19”
- Divide: solutions that start with flight sequence “17” and end in flight sequence “323”
- Factor: solutions that end with flight sequence “323”

Unary Multiplication

Cost	Slice	From/To	Date	day of dep.	next day	+ 2 days	+ 3 days	+ 4 days	+ 5 days	+ 6 days	+ 7 days	+ 8 days	+ 9 days
				mid	mid	mid	mid	mid	mid	mid	mid	mid	mid mid
$16.28	0	XXA to XXB	Sat-Mon, Nov 1-10	AA	AA	BB	BB	BB	BB	BB	BB	BB	BB
	1	XXB to XXC	Tue-Thur, Nov 11-20	SS	SS	II	II	II	II	II	II	II	II
	2	XXC to XXD	Fri-Sun, Nov 21-30	SS	SS	II	II	II	II	II	II	II	EE
	3	XXD to XXE	Mon-Wed, Dec 1-10	SS	SS	II	II	II	II	II	II	EE	EE
	4	XXE to XXF	Thur-Sat, Dec 11-20	SS	SS	II	II	II	II	II	EE	EE	EE
	5	XXF to XXG	Sun-Tue, Dec 21-30	SS	SS	II	II	II	II	EE	EE	EE	EE
	6	XXG to XXH	Wed-Fri, Dec 31-Jan 9	SS	SS	II	II	II	EE	EE	EE	EE	EE
	7	XXH to XXI	Sat-Mon, Jan 10-19	SS	SS	II	II	EE	EE	EE	EE	EE	EE
	8	XXI to XXJ	Tue-Thur, Jan 20-29	SS	SS	II	EE	EE	EE	EE	EE	EE	EE
	9	XXJ to XXA	Fri-Sun, Jan 30-Feb 8	EE	EE	EE	EE	EE	EE	EE	EE	EE	EE
$16.44	0	XXA to XXB	Sat-Mon, Nov 1-10	AA	AA	AA	AA	BB	BB	BB	BB	EE	EE
	1	XXB to XXC	Tue-Thur, Nov 11-20	SS	SS	SS	SS	II	II	II	II	EE	EE
	2	XXC to XXD	Fri-Sun, Nov 21-30	SS	SS	SS	SS	II	II	II	EE	EE	EE
	3	XXD to XXE	Mon-Wed, Dec 1-10	SS	SS	SS	SS	II	II	EE	EE	EE	EE
	4	XXE to XXF	Thur-Sat, Dec 11-20	SS	SS	SS	SS	II	EE	EE	EE	EE	EE
	5	XXF to XXG	Sun-Tue, Dec 21-30	EE	EE	EE	EE	EE	EE	EE	EE	EE	EE
	6	XXG to XXH	Wed-Fri, Dec 31-Jan 9	EE	EE	EE	EE	EE	EE	EE	EE	EE	EE
	7	XXH to XXI	Sat-Mon, Jan 10-19	EE	EE	EE	EE	EE	EE	EE	EE	EE	EE
	8	XXI to XXJ	Tue-Thur, Jan 20-29	EE	EE	EE	EE	EE	EE	EE	EE	EE	EE
	9	XXJ to XXA	Fri-Sun, Jan 30-Feb 8	EE	EE	EE	EE	EE	EE	EE	EE	EE	EE
$16.82	0	XXA to XXB	Sat-Mon, Nov 1-10	AA	AA	AA	AA	AA	AA	AA	AA	BB	BB
	1	XXB to XXC	Tue-Thur, Nov 11-20	SS	SS	SS	SS	SS	SS	SS	SS	II	II
	2	XXC to XXD	Fri-Sun, Nov 21-30	SS	SS	SS	SS	SS	SS	SS	SS	II	EE
	3	XXD to XXE	Mon-Wed, Dec 1-10	EE	EE	EE	EE	EE	EE	EE	EE	EE	EE
	4	XXE to XXF	Thur-Sat, Dec 11-20	EE	EE	EE	EE	EE	EE	EE	EE	EE	EE
	5	XXF to XXG	Sun-Tue, Dec 21-30	EE	EE	EE	EE	EE	EE	EE	EE	EE	EE
	6	XXG to XXH	Wed-Fri, Dec 31-Jan 9	EE	EE	EE	EE	EE	EE	EE	EE	EE	EE
	7	XXH to XXI	Sat-Mon, Jan 10-19	EE	EE	EE	EE	EE	EE	EE	EE	EE	EE
	8	XXI to XXJ	Tue-Thur, Jan 20-29	EE	EE	EE	EE	EE	EE	EE	EE	EE	EE
	9	XXJ to XXA	Fri-Sun, Jan 30-Feb 8	EE	EE	EE	EE	EE	EE	EE	EE	EE	EE

Binary Multiplication

Cost	Slice	From/To	Date	day of dep.	next day	+ 2 days	+ 3 days	+ 4 days	+ 5 days	+ 6 days	+ 7 days	+ 8 days
$18.36	0	XXA to XXB	Sat-Sun, Nov 1-9	A0	A1	A1	B1	S0	B1	S0	B0	S0
	1	XXB to XXC	Mon-Tue, Nov 10-18	A0	A1	B1	S0	B1	S1	B0	S1	S0
	2	XXC to XXD	Wed-Thur, Nov 19-27	A0	B1	S1	B1	S0	B0	S0	S1	S0
	3	XXD to XXA	Fri-Sat, Nov 28-Dec 6	B1	S0	B1	S1	B0	S0	S0	S1	S0
$18.63	0	XXA to XXB	Sat-Sun, Nov 1-9	A1	A1	A0	B0	S0	B1	S0	B1	S0
	1	XXB to XXC	Mon-Tue, Nov 10-18	A1	A1	B0	S0	B1	S0	B1	S0	S0
	2	XXC to XXD	Wed-Thur, Nov 19-27	A1	B0	S0	B1	S0	B1	S1	S1	S0
	3	XXD to XXA	Fri-Sat, Nov 28-Dec 6	B0	S0	B1	S1	B1	S0	S0	S1	S0
$20.45	0	XXA to XXB	Sat-Sun, Nov 1-9	A1	A0	A0	B1	S0	B0	S0	B1	S0
	1	XXB to XXC	Mon-Tue, Nov 10-18	A1	A0	B1	S0	B0	S0	B1	S0	S0
	2	XXC to XXD	Wed-Thur, Nov 19-27	A1	B1	S0	B0	S0	B1	S0	S0	S0
	3	XXD to XXA	Fri-Sat, Nov 28-Dec 6	B1	S0	B0	S1	B1	S0	S1	S0	S0
$20.54	0	XXA to XXB	Sat-Sun, Nov 1-9	A1	A0	A1	B1	S0	B0	S0	B0	S0
	1	XXB to XXC	Mon-Tue, Nov 10-18	A1	A0	B1	S0	B0	S1	B0	S0	S0
	2	XXC to XXD	Wed-Thur, Nov 19-27	A1	B1	S0	B0	S0	B0	S1	S0	S0
	3	XXD to XXA	Fri-Sat, Nov 28-Dec 6	B1	S0	B0	S1	B0	S0	S1	S0	S0
$21.37	0	XXA to XXB	Sat-Sun, Nov 1-9	A0	A1	A1	B1	S0	B1	S0	B1	S0
	1	XXB to XXC	Mon-Tue, Nov 10-18	A0	A1	B1	S0	B1	S1	B1	S1	S1
	2	XXC to XXD	Wed-Thur, Nov 19-27	A0	B1	S1	B1	S0	B1	S1	S0	S1
	3	XXD to XXA	Fri-Sat, Nov 28-Dec 6	B1	S0	B1	S1	B1	S0	S1	S0	S1
$21.73	0	XXA to XXB	Sat-Sun, Nov 1-9	A1	A1	A1	B0	S0	B1	S0	B1	S0
	1	XXB to XXC	Mon-Tue, Nov 10-18	A1	A1	B0	S0	B1	S0	B1	S1	S1
	2	XXC to XXD	Wed-Thur, Nov 19-27	A1	B0	S0	B1	S1	B1	S0	S0	S1
	3	XXD to XXA	Fri-Sat, Nov 28-Dec 6	B0	S0	B1	S1	B1	S0	S1	S0	S1

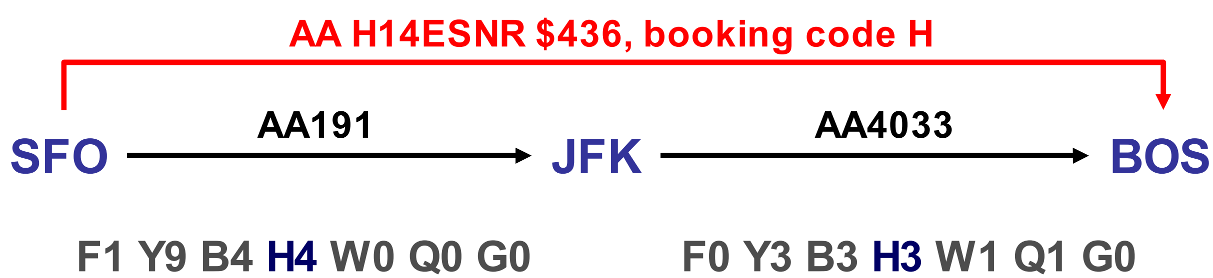
Outline

- Introduction
- Flights
- How airline prices work
- Complexity of travel planning
- Demos
- **Seat Availability**
- Further Reading

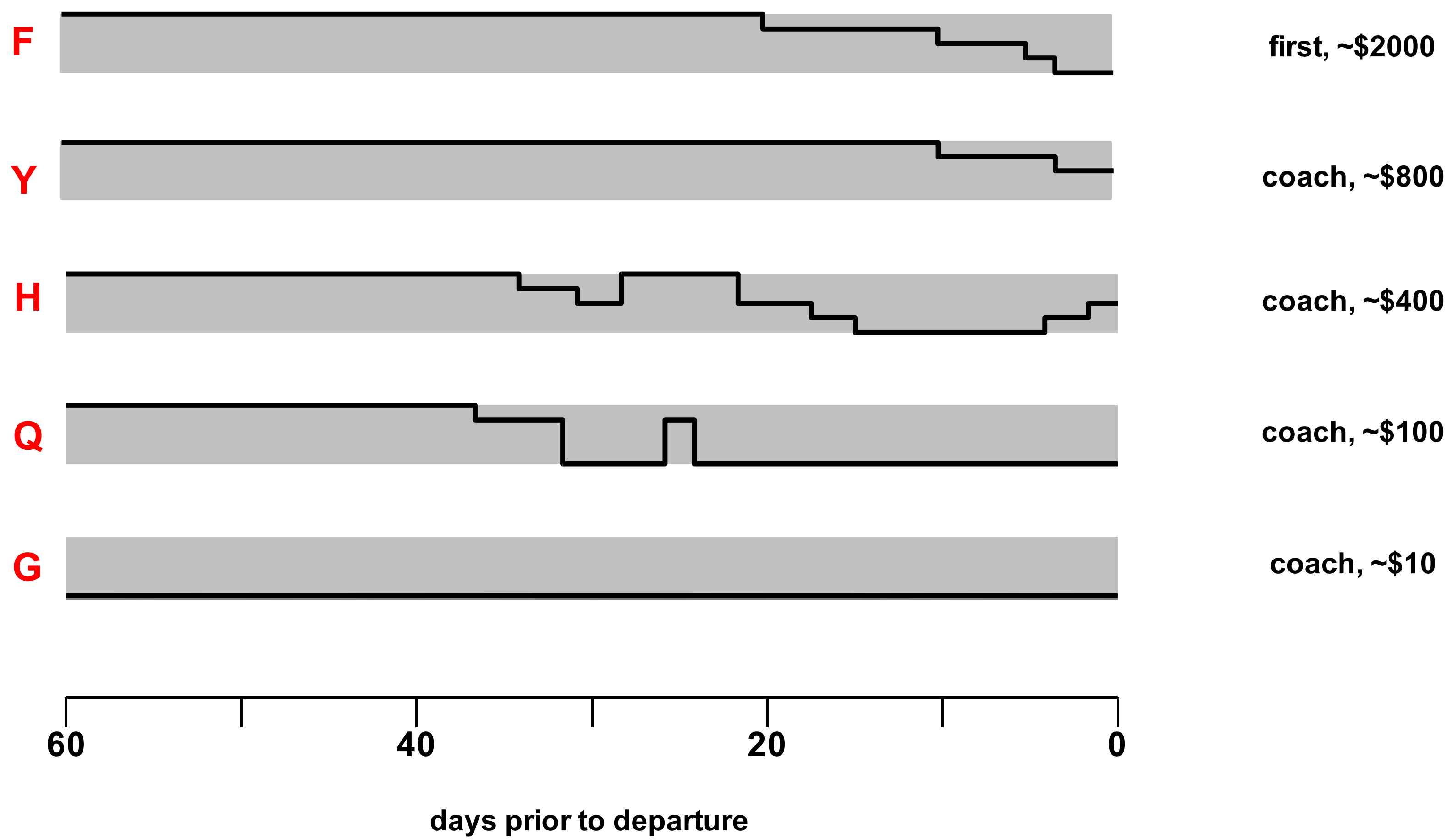
Seat Availability

How many seats free on AA191 SFO-JFK, April 2 ?

- Airlines use seat availability to adjust prices according to demand
- Every fare is assigned a *booking code* (F, Y, B, H, Q, …), based on price and cabin
  - usually first letter of fare basis code
- Availability of seat is dependent on booking code purchased

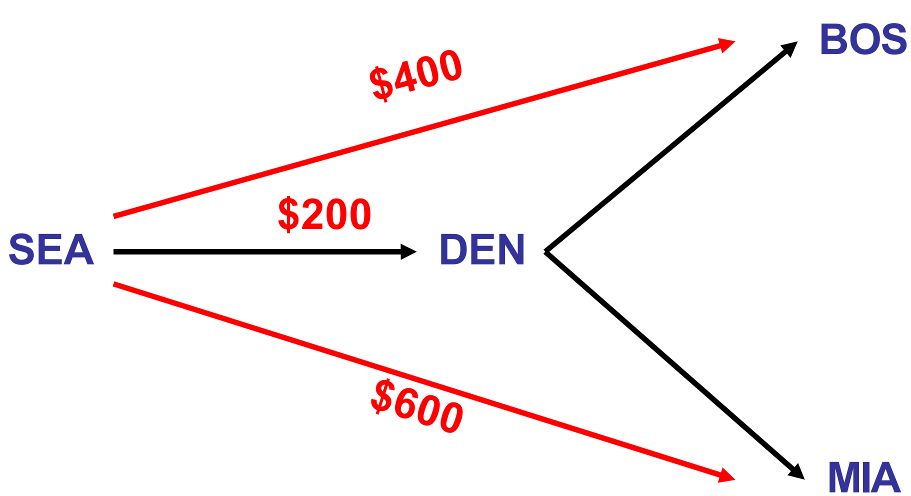


Availability Dynamics



O & D Availability

- To the airline, each seat is a potential part of many products
  – different products compete for seat
  – query must provide not just flight and cabin, but product context
    - trip origin & destination (O&D)
    - future: frequent flyer number, Swiss bank account #, etc
  – very difficult optimization problem for airline



UA131 SEA-DEN JUNE 10		
Trip O	Trip D	Availability
SEA	DEN	F3 Y9 W2 Q0
SEA	BOS	F2 Y9 W1 Q0
SEA	MIA	F3 Y9 W5 Q3

Seat Availability

- 1 plane/sec · 150 psgr/plane · 100 search/psgr · 1000 fl/search = 15,000,000 availability questions per second
  - airline computers can't support this load
  - airline networks can't support this load
  - ITA Software uses distributed, scalable cache

- Airline would like to take more features of trip into account
  - all flights; all passengers; total price; etc
  - would be disastrous for search: too many questions to ask

- No locking: answer is not guaranteed for any period of time
  - between search and purchase, availability may have changed

Further Information

- Unfortunately, this is not an area with a big published literature.
  - Large academic and industry literature on optimization problems like setting prices and routes and seat availability
  - But no work covers search from a consumer perspective, or covers complexity
  - There is no nice problem statement
    - The problem is defined mostly by IATA (International Air Transport Association, a cartel of airlines) and ATP (Airline Tariff Publishing Company, manager of electronic fare and rule formats), but they provide no formal specifications
  - The problem statement and results I've presented here are mine
    - Unpublished and not common knowledge
- Further reading
  - Introductory chapters of MS/PhD theses on revenue management
  - Other academic/industry literature on revenue management and schedule optimization
  - “Hard Landing”, by Thomas Petzinger – very colorful history of airlines

Exercise

E-mail this itinerary | Back to search results | Modify search | Log out | Comments | Help | ITA Software

$1717.26 in US Dollars
1 adult @ $1717.26

Buy it!

Hide booking details

Debug solution

This ticket is non-refundable.

Changes to this ticket will incur a penalty fee.

Airport maps/services:
BOS: Boston Logan
DTW: Detroit Wayne County
MSP: Minneapolis/St. Paul Int'l
HNL: Honolulu Int'l
LAX: Los Angeles Int'l
PWM: Portland Int'l

Boston, MA to Honolulu, HI: 5121 miles — 14 hrs 49 min

Northwest Airlines Flight NW1821 on an Airbus A-319 (jet) in coach class
Departs **Boston, MA** (BOS) Sat, Sept 13 **6:00a** 2 hrs 3 min
Arrives **Detroit, MI** (DTW) **8:03a**
1 adult in booking code M, covered by fare (A1) below
avail checked(live): B9 F9 H9 K9 L9 M9 P9 Q9 T9 V9 Y9; strict-local

Layover in Detroit — 1 hr 2 min

Northwest Airlines Flight NW763 on a Boeing B-757 (jet) in coach class
Departs **Detroit, MI** (DTW) Sat, Sept 13 **9:05a** 1 hr 55 min
Arrives **Minneapolis/Saint Paul, MN** (MSP) **10:00a**
1 adult in booking code Q, covered by fare (B1) below
avail checked(live): B9 F9 H9 M9 P9 Q9 V9 Y9 (married: NW763,NW921); strict-o&d

Layover in Minneapolis/Saint Paul — 1 hr 30 min

Northwest Airlines Flight NW921 on a McD-Douglas DC-10 (jet) in coach class (lunch, snack)
Departs **Minneapolis/Saint Paul, MN** (MSP) Sat, Sept 13 **11:30a** 8 hrs 19 min
Arrives **Honolulu, HI** (HNL) **2:49p**
1 adult in booking code Q, covered by fare (B1) below
avail checked(live): B9 F0 H9 M9 P9 Q9 V9 Y9 (married: NW763,NW921); strict-o&d

Honolulu, HI to Los Angeles, CA: 2552 miles — 5 hrs 2 min

Northwest Airlines Flight NW930 on a McD-Douglas DC-10 (jet) in coach class (dinner)
Departs **Honolulu, HI** (HNL) Sat, Sept 20 **4:48p** 5 hrs 2 min
Arrives **Los Angeles, CA** (LAX) Sun, Sept 21 **12:50a**
1 adult in booking code V, covered by fare (B2) below
avail checked(live): B9 F4 H9 K9 L9 M9 P9 Q9 T9 V9 Y9; pseudo-o&d

Los Angeles, CA to Portland, ME: 2640 miles — 6 hrs 52 min

Northwest Airlines Flight NW334 on a Boeing B-757 (jet) in coach class (lunch)
Departs **Los Angeles, CA** (LAX) Wed, Oct 1 **12:35p** 4 hrs 19 min
Arrives **Detroit, MI** (DTW) **7:54p**
1 adult in booking code V, covered by fare (B2) below
avail checked(live): B9 F9 H9 K0 L9 M9 P9 Q9 T9 V9 Y9; pseudo-o&d

Layover in Detroit — 42 min

Northwest Airlines Flight NW3468 on an Avro RJ (jet) in coach class (operated by Mesaba Aviation)
Departs **Detroit, MI** (DTW) Wed, Oct 1 **8:36p** 1 hr 51 min
Arrives **Portland, ME** (PWM) **10:27p**
1 adult in booking code V, covered by fare (B2) below
avail checked(live): B9 F9 H9 K9 L9 M9 P9 Q9 T9 V9 Y9; pseudo-o&d

Note: The layover in Detroit (DTW) has relatively little room for delays, and for this route a missed connection would likely be very inconvenient.

Portland, ME to Boston, MA: 1296 miles — 6 hrs 15 min

Northwest Airlines Flight NW5872 on a Canadair Reg. Jet (jet) in coach class (operated by Express Airlines)
Departs **Portland, ME** (PWM) Tue, Oct 7 **6:06a** 2 hrs 13 min
Arrives **Detroit, MI** (DTW) **8:19a**
1 adult in booking code M, covered by fare (B3) below
avail checked(live): B9 H9 K9 L9 M9 Q9 T9 V9 Y9; strict-local

Layover in Detroit — 2 hrs 16 min

Northwest Airlines Flight NW336 on a Boeing B-757 (jet) in coach class
Departs **Detroit, MI** (DTW) Tue, Oct 7 **10:35a** 1 hr 46 min
Arrives **Boston, MA** (BOS) **12:21p**
1 adult in booking code M, covered by fare (A2) below
avail checked(live): B9 F9 H9 K9 L9 M9 P9 Q9 T9 V9 Y9; strict-local

Booking details

Buying this ticket online using our website is the easiest and most reliable way to obtain this ticket at this price. However, if we are unable to sell or you don't want to buy the ticket online, the information on this page will enable you to buy the ticket from the airline (Northwest Airlines: 1-800-225-2525, http://www.nwa.com/) or a travel agent. If you use a travel agent to buy this ticket:

- If your travel agent is online and has an e-mail address, e-mail this itinerary to them
- If your travel agent is not online, print out this page and fax/give it to them

It is very important to use the exact same booking codes and fare codes that we've used on this page in order to match the price we've found.

Fare (A1):	**NW BOS==>DTT ME7NR fare (round trip fare)**	$250.23
	Tax: US Transportation Tax (US)	$18.77
Fare (B1):	**NW DTT==>HNL QLWE7N fare (round trip fare)**	$416.02
	Tax: US Transportation Tax (US)	$14.61
Fare (B2):	**NW HNL==>PWM VLW7EN fare (round trip fare)**	$433.43
	Tax: US Transportation Tax (US)	$17.30
Fare (B3):	**NW PWM==>DTT ME7NR fare (round trip fare)**	$228.37
	Tax: US Transportation Tax (US)	$17.13
Fare (A2):	**NW DTT==>BOS ME7NR fare (round trip fare)**	$250.23
	Tax: US Transportation Tax (US)	$18.77
Tax:	US Alaska/Hawaii Departure Tax (US)	$13.40
Tax:	US Flight Segment Tax (ZP)	$24.00
Tax:	US Passenger Facility Charge (XF)	$15.00

Total for 1 adult passenger: **$1717.26**

(as of Wednesday, September 3, 2003 2:45am; fares loaded Tuesday, September 2, 2003 8:33pm)

Fare calc:
BOS NW DTT Q9.30 240.93ME7NR NW X/MSP NW HNL Q9.30 406.72QLWE7N NW LAX S55.81 NW X/DTT NW PWM Q9.30 368.32VLW7EN NW DTT Q9.30 219.07ME7NR NW BOS Q9.30 240.93ME7NR USD 1578.28 END SITI XT 99.98US ZP 24.00DTW XF 15.00DTW

Priceable units:
Fares A1, A2: round trip
Fares B1, B2, B3: circle trip

POWERED BY ita

This is a solution as displayed on the ITA Software web site, one of 2,197,704,882,975,408 the ITA Software search engine found for a BOS-HNL-LAX-PWM-BOS circle query, with one-day departure windows for each part of the trip

- How much of this output can you understand now?
- Draw the trip with fares, priceable units and booking codes